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A COMPUTER PROGRAM FOR FITTING CENSORED SAMPLES TO A WEIBULL DI--ETC(U)

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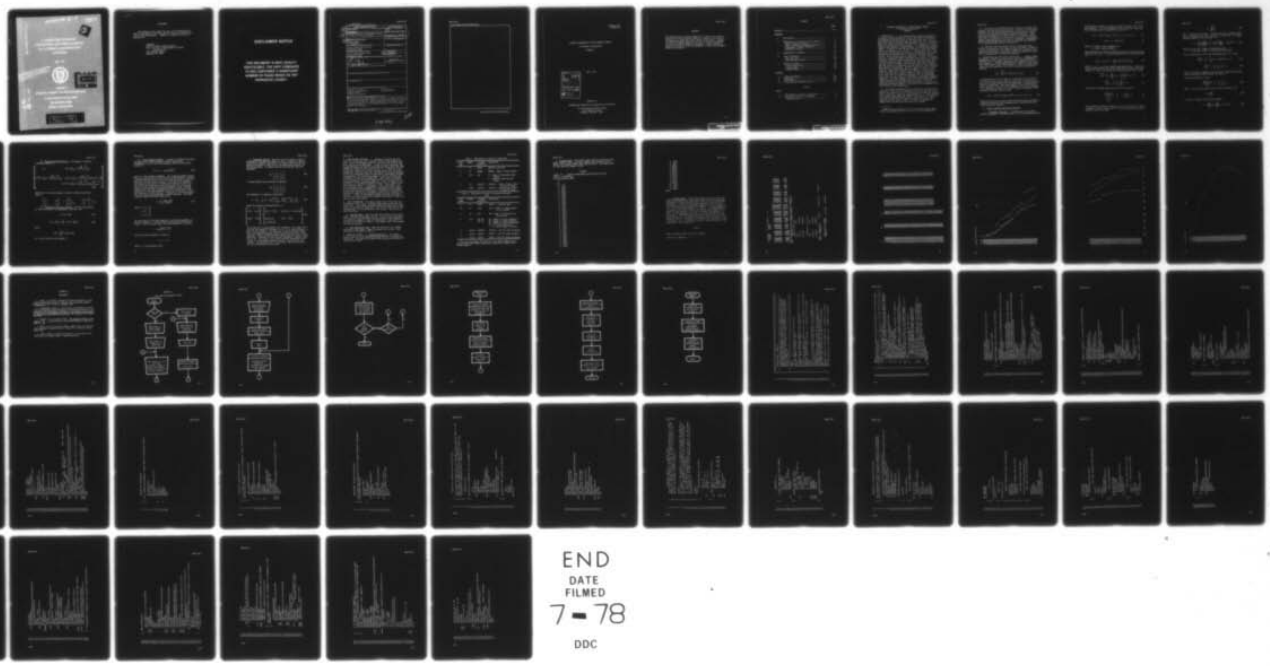

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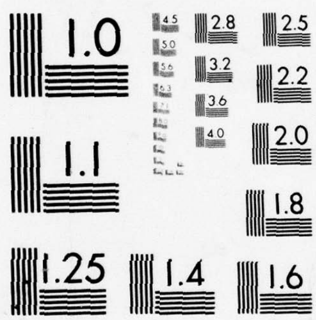

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A COMPUTER PROGRAM FOR FITTING CENSORED SAMPLES TO A WEIBULL DISTRIBUTION (CENWEIB)

APRIL 1978

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

14

1. REPORT NUMBER: CAA-D-78-5

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

9

4. TITLE (and Subtitle):

6

A Computer Program for Fitting Censored Samples to a Weibull Distribution (CENWEIB).

5. TYPE OF REPORT & PERIOD COVERED: Documentation rept.

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s):

10

Mr. Jerry Thomas
Cadet John Gallo

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS:

US Army Concepts Analysis Agency
8120 Woodmont Avenue
Bethesda, MD 20014

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS:

US Army Concepts Analysis Agency
8120 Woodmont Avenue
Bethesda, MD 20014

11

12. REPORT DATE: April 1978

13. NUMBER OF PAGES: 64

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

12 59p.

15. SECURITY CLASS. (of this report): UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Weibull distribution
Censored-uncensored samples
Computer program
Progressively censored
Singly censored
Estimating equations

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

1/B

390 996

CAA-D-78-5

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

DOCUMENTATION
CAA-D-78-5

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(CENWEIB)

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ABSTRACT

The Censored Weibull Program (CENWEIB) is a UNIVAC 1108 computer program for fitting censored samples to a Weibull distribution or for generating random data from a Weibull distribution. Maximum likelihood estimating equations are used to calculate parameters for the two-parameter Weibull distribution. Moment estimating equations are used to calculate parameters for the three-parameter Weibull distribution.

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TABLES

A COMPUTER PROGRAM FOR FITTING CENSORED SAMPLES TO A WEIBULL DISTRIBUTION (CENWEIB)

1. INTRODUCTION. a. The Censored Weibull Program (CENWEIB)* is a computer program designed for use on the UNIVAC 1108 computer. The program can perform either of two specified tasks. First, given the scale (α) and the shape (β) parameters, it can generate data from a specified Weibull distribution. Secondly, it can fit a Weibull distribution to observed data. The program can fit data obtained from complete, singly-censored, or progressively-censored samples. Examples of data are the failure of a life-test component or the detection of a target in a time-to-detect test. Censored data are obtained when units are removed from a test before the event to be measured (e.g., failure or detection) has occurred. There are two types of censoring. In Type I censoring, testing is terminated after a specified time interval. In Type II censoring, testing is terminated after the specified event occurs a certain number of times. In both cases, the data collected consists of the termination times of the events which occurred, the time testing is terminated, and the number of items remaining when testing is terminated. When all remaining tests are terminated at one specified time, the test sample is called singly-censored. Some tests, however, are conducted so that at the initial censoring only a certain number of items are terminated while some items are allowed to continue until the designated event occurs or until they are terminated at subsequent stages of censoring. Such test samples are called progressively-censored. CENWEIB can be used for either Type I or Type II censoring, singly-censored or progressively-censored samples, as well as on complete (uncensored) samples. The program can accommodate up to and including 1,000 observations.

b. The Weibull distribution can approximate a variety of distributions. When the shape parameter (discussed below) of the Weibull distribution is 1.0, the distribution becomes the exponential distribution. When the shape parameter is 3.5, the distribution closely approximates the normal distribution. The Weibull distribution can also take on various other positively- and negatively-skewed forms. CENWEIB is primarily designed for samples which are believed to be positively skewed.

---

*CENWEIB was initiated by Carl B. Bates (CAA) and was formulated and programed by Keith D. Thorp, a former CAA employee.

c. The basis of the CENWEIB Program is the work of Cohen (reference 1) and Essenwanger (reference 2). Cohen's scale and shape parameters for the two-parameter Weibull distribution are calculated by using maximum likelihood estimating equations. Simple iterative techniques may be used to derive the required values. Once Cohen's parameters are found, they are converted to the more familiar and conventional form used by Essenwanger. Essenwanger's moment estimating equations are used to calculate the parameters of the three-parameter Weibull distribution.

d. When more than one sample of data is generated from a Weibull distribution or more than one sample of data is fitted to a Weibull distribution, the CENWEIB Program will calculate a Chi-square statistic which can be used to test the statistical equality of the generated or fitted distributions.

2. MATHEMATICAL AND STATISTICAL DESCRIPTION. a. Weibull Sample Generation. CENWEIB uses the Weibull cumulative distribution function (cdf) to randomly generate Weibull numbers from given input parameters. Uniform random numbers are generated from a uniform random number generating subroutine. After a uniform random number has been generated, the Weibull cumulative distribution function

$$F(x) = \int_0^x \beta\, t^{\beta-1}\exp(-t/\alpha)\, dt/\alpha^{\beta}, \qquad [1]$$

is integrated from 0 to x, where x is the point on the distribution that defines an area under the curve equal to the value of the uniform random number which was generated. N of these Weibull values are generated. Then using these N values CENWEIB calculates estimates ($\hat{\alpha}$ and $\hat{\beta}$) of the parameters (α and β) of equation [1]. Substituting $\hat{\alpha}$ and $\hat{\beta}$ for α and β, gives an estimate of the probability density function (pdf)

$$f(x) = (\beta/\alpha^{\beta})x^{\beta-1}\exp[-(x/\alpha)^{\beta}],\ x \geq 0, \alpha > 0, \beta > 0. \qquad [2]$$

Random variation may actually cause the true Weibull distribution parameters produced by the generated data to be different from those originally inputted.

b. Cohen's Maximum Likelihood Estimation

(1) Parameter Estimation. The method used to estimate Weibull parameters in the development of the two-parameter Weibull

distribution in CENWEIB is Cohen's maximum likelihood (ML) estimation technique. In the development of this technique, Cohen uses the following forms of the Weibull pdf and cdf:

$$f(x) = (\gamma/\theta)x^{\gamma-1}\exp(-x^{\gamma}/\theta);\ x \geq 0, \gamma > 0, \theta > 0, \tag{3}$$

$$F(x) = 1 - \exp(-x^{\gamma}/\theta), \tag{4}$$

where θ is Cohen's scale parameter and
γ is Cohen's shape parameter.

Cohen develops his parameter estimating equations from the maximum likelihood function. For complete samples the likelihood function is:

$$L(x_1,\ldots,x_n;\gamma,\theta) = \prod_{i=1}^{n}(\gamma/\theta)x_i^{\gamma-1}\exp(-x_i^{\gamma}/\theta), \tag{5}$$

where $x_1, x_2,\ldots,x_n$ are all uncensored observations. Taking the partial derivatives of the natural logarithm of equation [5] with respect to γ and θ and setting the results equal to zero, we get:

$$\frac{\partial \ln L}{\partial \gamma} = \frac{n}{\gamma} + \sum_{i=1}^{n}\ln x_i - \frac{1}{\theta}\sum_{i=1}^{n}x_i^{\gamma}\ln x_i = 0, \tag{6}$$

$$\frac{\ln L}{\partial \theta} = -\frac{n}{\theta} + \frac{1}{\theta^2}\sum_{i=1}^{n}x_i^{\gamma} = 0. \tag{7}$$

Eliminating θ between equations [6] and [7] we obtain:

$$\frac{\sum_{i=1}^{n} x_i^{\gamma}\ln x_i}{\sum_{i=1}^{n} x_i^{\gamma}} - \frac{1}{\gamma} = \frac{1}{n}\sum_{i=1}^{n}\ln x_i. \tag{8}$$

Using standard iterative procedures, this can be solved for the ML estimate $\hat{\gamma}$. After solving for $\hat{\gamma}$, $\hat{\theta}$ can be determined using equations [6] and [7] so that

$$\hat{\theta} = \sum_{i=1}^{n} \frac{x_i^{\hat{\gamma}}}{n} \qquad [9]$$

The "^" denotes an estimator. Maximum likelihood estimating equations are analogous for Type I and Type II censoring. For singly-censored samples, the ML function is:

$$L = \frac{N!}{(N-n)!}\left[\prod_{i=1}^{n} \frac{\gamma}{\theta}\, x_i^{\gamma-1} \exp\left(-\frac{x_i^{\gamma}}{\theta}\right)\right][1 - F(x_T)]^{N-n}, \qquad [10]$$

where N is the total number of observations and
n is the total number of uncensored observations.

$F(x_T)$ is the Weibull cdf at the termination point, x_T. Then,

$$\frac{\partial \ln L}{\partial \gamma} = \frac{n}{\gamma} + \sum_{i=1}^{n} \ln x_i - \frac{1}{\theta}\sum{}^{*} x_i^{\gamma} \ln x_i = 0, \qquad [11]$$

$$\frac{\partial \ln L}{\partial \theta} = -\frac{n}{\theta} + \frac{1}{\theta^2}\sum{}^{*} x_i^{\gamma} = 0, \qquad [12]$$

where $\sum^{*}$ denotes a summation over the whole sample with the censored observations being assigned the value x_T.

From these equations, we get

$$\frac{\sum^{*} x_i^{\gamma} \ln x_i}{\sum^{*} x_i^{\gamma}} - \frac{1}{\gamma} = \frac{1}{n}\sum_{i=1}^{n} \ln x_i. \qquad [13]$$

Again, $\hat{\gamma}$ can be calculated using iterative techniques, and

$$\hat{\theta} = \sum{}^{*} \frac{x_i^{\hat{\gamma}}}{n}. \qquad [14]$$

For Type I progressively-censored samples,

$$L = C \prod_{i=1}^{n} f(x_i) \prod_{i=1}^{k} [1 - F(T_i)]^{r_i} \qquad [15]$$

where C is a constant,

k is the number of times censoring occurred,

T_i $(i = 1,\ldots,k)$ are the times of censoring, and

r_i is the number of observations randomly terminated at the i^{th} stage of censoring. Then,

$$\frac{\partial \ln L}{\partial \gamma} = \frac{n}{\gamma} + \sum_{i=1}^{n} \ln x_i - \frac{1}{\theta}\sum^{**} x_i^{\gamma} \ln x_i = 0, \qquad [16]$$

$$\frac{\partial \ln L}{\partial \theta} = -\frac{n}{\theta} + \frac{1}{\theta^2}\sum^{**} x_i^{\gamma} = 0, \qquad [17]$$

where $\sum^{**}$ denotes summation over all the observations, with an observation censored at time, T_i, assigned the value T_i.

We can then derive:

$$\frac{\sum^{**} x_i^{\gamma} \ln x_i}{\sum^{**} x_i^{\gamma}} - \frac{1}{\gamma} = \frac{1}{n}\sum_{i=1}^{n} \ln x_i. \qquad [18]$$

After determining $\hat{\gamma}$,

$$\hat{\theta} = \sum^{**} x_i^{\gamma}/n. \qquad [19]$$

The two estimating equations for the Type II progressively-censored samples are analogous to equations [18] and [19] although intermediate steps in their derivation differ. As can be seen from the above, the likelihood equations for each of the three censoring cases (complete, singly-, and multiply-censored) are different. However, the parameter estimating equations derived from maximum likelihood equations are basically the same. This enables us to use one equation, each with the appropriate summation, to estimate the scale and shape parameters.

(2) Variance-Covariance Matrix. Cohen's variance-covariance matrix is approximated by using the estimated values of the parameters to construct the information matrix. The inverse of the information matrix is the variance-covariance matrix. The approximate variance-covariance matrix is thus:

$$\begin{bmatrix} -\dfrac{\partial^2 \ln L}{\partial \gamma^2}\Big|_{\hat{\gamma},\hat{\theta}} & -\dfrac{\partial^2 \ln L}{\partial \gamma \partial \theta}\Big|_{\hat{\gamma},\hat{\theta}} \\ -\dfrac{\partial^2 \ln L}{\partial \theta \partial \gamma}\Big|_{\hat{\gamma},\hat{\theta}} & -\dfrac{\partial^2 \ln L}{\partial \theta^2}\Big|_{\hat{\gamma},\hat{\theta}} \end{bmatrix}^{-1} = \begin{bmatrix} V(\hat{\gamma}) & Cov(\hat{\gamma},\hat{\theta}) \\ Cov(\hat{\gamma},\hat{\theta}) & V(\hat{\theta}) \end{bmatrix}. \quad [20]$$

Calculation of the second partials of the complete sample ML function for the information matrix gives:

$$-\frac{\partial^2 \ln L}{\partial \gamma^2}\Big|_{\hat{\gamma},\hat{\theta}} = \frac{n}{\hat{\gamma}^2} + \frac{1}{\hat{\theta}} \sum_{i=1}^{n} x_i^{\hat{\gamma}}(\ln x_i)^2, \quad [21]$$

$$-\frac{\partial^2 \ln L}{\partial \gamma \partial \theta}\Big|_{\hat{\gamma},\hat{\theta}} = -\frac{1}{\hat{\theta}^2} \sum_{i=1}^{n} x_i^{\hat{\gamma}} \ln x_i, \quad [22]$$

$$-\frac{\partial^2 \ln L}{\partial \theta^2}\Big|_{\hat{\gamma},\hat{\theta}} = -\frac{n}{\hat{\theta}^2} + \frac{2}{\hat{\theta}^3} \sum_{i=1}^{n} x_i^{\hat{\gamma}}. \quad [23]$$

The second partials of the ML functions for the singly- and progressively-censored samples can be taken in a similar manner to obtain results analogous to equations [21], [22] and [23].

c. Essenwanger's Moment Estimation

(1) Parameter Estimation. In Essenwanger's more conventional notation, the Weibull pdf and cdf are:

$$f(x) = (\beta/\alpha^\beta)x^{\beta-1}\exp[-(x/\alpha)^\beta]; \; x \geq 0, \alpha > 0, \beta > 0 \quad [24]$$

$$F(x) = 1 - \exp(-x/\alpha)^\beta \quad [25]$$

where α is Essenwanger's scale parameter and
β is Essenwanger's shape parameter.

CENWEIB calculates Weibull parameters based on Cohen's form of the Weibull distribution and converts to Essenwanger's form using the relationships $\beta = \gamma$ and $\alpha = \theta^{1/\beta}$. In other words, Cohen's and Essenwanger's shape parameters are the same, but the scale parameters are different, although related to each other. CENWEIB outputs Essenwanger's parameters.

(2) Variance-Covariance Matrix. Essenwanger's variance-covariance matrix is:

$$\begin{bmatrix} c_{11} & \left(-\alpha^{\beta}\ln\frac{1}{\beta}\right)c_{11} + \left(\frac{\alpha^{\left(\frac{1}{\beta}-1\right)}}{\beta}\right)c_{12} \\ \left(-\alpha^{\beta}\ln\frac{1}{\beta}\right)c_{11} + \left(\frac{\alpha^{\left(\frac{1}{\beta}-1\right)}}{\beta}\right)c_{12} & \left[\left(-\alpha^{\beta}\ln\frac{1}{\beta}\right)c_{11} + \left(\frac{\alpha^{\left(\frac{1}{\beta}-1\right)}}{\beta}\right)c_{12}\right]\left[-\alpha^{\beta}\ln\frac{1}{\beta}\right] \\ & + \left[\left(-\alpha^{\beta}\ln\frac{1}{\beta}\right)c_{12} + \left(\frac{\alpha^{\left(\frac{1}{\beta}-1\right)}}{\beta}\right)c_{22}\right]\left[\frac{\alpha^{\left(\frac{1}{\beta}-1\right)}}{\beta}\right] \end{bmatrix} \quad [26]$$

where the c's are the elements of Cohen's variance-covariance matrix:

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} V(\hat{\gamma}) & Cov(\hat{\gamma},\hat{\theta}) \\ Cov(\hat{\gamma},\hat{\theta}) & V(\hat{\theta}) \end{bmatrix} \quad [27]$$

(3) Distribution Mean and Variance. The mean and variance of the distribution, in Essenwanger's notation, are:

$$\mu = \alpha\Gamma\left(1 + \frac{1}{\beta}\right), \quad [28]$$

$$\sigma^2 = \alpha^2\left(\Gamma\left(1 + \frac{2}{\beta}\right) - \left[\Gamma\left(1 + \frac{1}{\beta}\right)\right]^2\right), \quad [29]$$

where

$$\Gamma(x) = \int_0^{\infty} t^{x-1}\exp(-t)dt$$

for a given value of the argument x.

(4) Three-parameter Weibull. A segment of CENWEIB calculates parameters for a three-parameter Weibull distribution. Essenwanger's form of the three-parameter Weibull distribution function is

$$F(x) = 1 - \exp[-(\frac{x-\delta}{\alpha})^{\beta}], \qquad [30]$$

where δ is the location parameter. The location parameter identifies the starting point of the function on the abscissa. The two-parameter Weibull form will force the starting point of the function to be at zero on the abscissa. Since the three-parameter form does not force this, it allows the fitting of a curve to a set of data which may more closely approximate the data values than the curve produced from the two-parameter form. Although CENWEIB plots only the two-parameter Weibull distribution, the parameters of the three-parameter Weibull distribution are also in the output as an aid to the user. Estimates of the three parameters are found using Essenwanger's moment technique. The shape parameter moment estimating equation is

$$B = \frac{c - 3ab + 2a^3}{(b - a^2)^{3/2}}, \qquad [31]$$

where $a = \Gamma(1 + \frac{1}{\beta})$

$b = \Gamma(1 + \frac{2}{\beta})$

$c = \Gamma(1 + \frac{3}{\beta})$

As can be seen in the above equation, β, the shape parameter, is the only unknown. It can be found by employing iterative techniques. Once β has been found, the scale parameter is given as

$$\alpha = \sqrt{\sigma^2/(b - a^2)}$$

and the location parameter is given as

$$\delta = \mu - a\,\alpha$$

where μ is the population mean.

d. Hypothesis Testing. When more than one sample of data is generated or more than one sample of data is fitted to a Weibull distribution, CENWEIB calculates a Chi-square statistic with two degrees of freedom in order to test the statistical equality of the distributions. To compare the two distributions, the null hypothesis

$$H_0: \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix}, \qquad [32]$$

is tested against the alternative hypothesis

$$H_a: \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix} \neq \begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix}. \qquad [33]$$

The hypothesis is tested by calculating

$$Q = [\hat{\alpha}_1 - \hat{\alpha}_2, \hat{\beta}_1 - \hat{\beta}_2] \begin{bmatrix} \sigma^2(\hat{\alpha}) & \sigma(\hat{\alpha},\hat{\beta}) \\ \sigma(\hat{\alpha},\hat{\beta}) & \sigma^2(\hat{\beta}) \end{bmatrix}^{-1} \begin{bmatrix} \hat{\alpha}_1 - \hat{\alpha}_2 \\ \hat{\beta}_1 - \hat{\beta}_2 \end{bmatrix}, \qquad [34]$$

where the variance-covariance matrix is

$$\begin{bmatrix} \sigma^2(\hat{\alpha}) & \sigma(\hat{\alpha},\hat{\beta}) \\ \sigma(\hat{\alpha},\hat{\beta}) & \sigma^2(\hat{\beta}) \end{bmatrix} = \begin{bmatrix} V(\hat{\alpha}_1) + V(\hat{\alpha}_2) & Cov(\hat{\alpha}_1,\hat{\beta}_1) + Cov(\hat{\alpha}_2,\hat{\beta}_2) \\ Cov(\hat{\alpha}_1,\hat{\beta}_1) + Cov(\hat{\alpha}_2,\hat{\beta}_2) & V(\hat{\beta}_1) + V(\hat{\beta}_2) \end{bmatrix}. \qquad [35]$$

The Q-statistic is approximately distributed as a Chi-square variate with two degrees of freedom, i.e., $Q \sim \chi^2(2)$ (see references 3 and 4 for more detail). An inspection of the Q-statistic shows that close agreement between the two distributions yields a small statistic, while a large difference between the two yields a large statistic. Therefore, to test the null hypothesis, compare Q with $\chi^2(1-\alpha, 2)$. If $Q \geq \chi^2(1-\alpha, 2)$, reject the null hypothesis at the α-level of significance; otherwise, do not reject the null hypothesis. By rejecting the null hypothesis, we are saying that the two distributions are not equal.

3. COMPUTATIONAL PROCEDURE. a. Appendix D contains the flowchart for the program and a complete listing of the computer program. First, the program checks to see if data are to be fitted or if data are to be generated. If data are to be generated, the program generates the required data with specified scale and shape parameters, orders the data, and censors it. If data are to be fitted, the program first orders the data, then calls the WEIBUL subroutine. Using the data, this subroutine calculates parameters for the three-parameter Weibull distribution using moment estimating equations. An iterative procedure is used to calculate the parameters if there are censored data. Then, Cohen's parameters are calculated for the two-parameter Weibull distribution using maximum likelihood estimating equations. Again, if there are censored data, an iterative procedure is used. First, Cohen's variance-covariance matrix is calculated. Then, from Cohen's variance-covariance matrix, Essenwanger's variance-covariance matrix is calculated and the parameters for Essenwanger's two-parameter Weibull distribution are calculated. Using the PLOT subroutine, the Weibull cumulative and probability distribution functions are plotted. If there is more than one sample of data, Chi-square statistics are then calculated to compare each sample of data with each previous data sample.

4. INPUT PREPARATION. The deck of input cards follows the program EXECUTE (@XQT) card. A FINISH (@FIN) card is the last card in the deck. The program terminates when the FINISH card is read. There are two types of input deck: one for data sets and one for use when data is to be generated by the program. Both types are described below.

a. Data Set Input. Data set input must consist of four card types. The data are grouped; that is, if two or more data values are identical, the data value is given along with the frequency, indicating the number of identical data values. Each type of input card is described in Table 1. Card types 3 and 4 are repeated for multiple samples.

b. Data Generation Input. When the data are to be randomly generated by the program, seven card types must be used. Each card type is described in Table 2.

5. NUMERICAL EXAMPLE. a. Problem Description. The example problem consists of 68 completed observations and 32 censored observations. The sample was progressively censored with three different censor times.

Table 1. Description of Cards for Input Data

Card type	Format	Program variable(s)	Explanation
1	I5	IDATA	IDATA=0, read data
2	I5	NRSAMP	NRSAMP = number of data samples
3	2I5	N,K	N = number of uncensored data groups K = number of censored data groups
4	I5,	JFREQ(I),	JFREQ(I) = number of like values in the i^{th} group
	F10.5	VALUE(I)	VALUE(I) = common value of data in the i^{th} group

Table 2. Description of Cards for Randomly Generated Data

Card number	Format	Program variable(s)	Explanation
1	I5	IDATA	IDATA = 1, generate data
2[a]	I5	IZZQ	IZZQ = number of random numbers to be skipped
3	I5	INO	INO = number of data points to be generated
4	4I5	NA, NB, NC, NS	NA = number of scale parameters NB = number of shape parameters NC = number of censor percentages NS = 0, uniform censoring = 1, censor upper 75 percent
5	6F10.2	ALPHA(I)	ALPHA(I) = the i^{th} scale parameter
6	6F10.2	BETA(I)	BETA(I) = the i^{th} shape parameter
7	6F10.2	CEN(I)	CEN(I) = the i^{th} censor percentage

[a]This card eliminates the need for a new random number seed in the uniform random number generating subroutine by skipping IZZQ random numbers.

b. <u>Program Input</u>. The program input consists of the four card types listed in Table 1 and four systems cards required by the UNIVAC 1108 computer (@RUN, @ASG, @XQT and @FIN). The input cards for the numerical example follow.

Example

```
@RUN,/TP     A106A,F1830A8397B,UNCLASSIFIED,2,200
@ASG,A   06WEIBULL.
@XQT 06WEIBULL.RUN
   0
   1
  47    3
   5  177.
   1  246.
   1  252.
   1  269.
   1  283.
   1  294.
   5  331.
   1  367.
   1  379.
   1  386.
   1  411.
   1  423.
   1  441.
   1  488.
   1  502.
   1  508.
   1  519.
   1  531.
   1  542.
   1  550.
   1  553.
   1  568.
   1  583.
   1  589.
   1  601.
   1  613.
   1  621.
   5  682.
   1  772.
   1  806.
   1  820.
   1  840.
   1  854.
   1  872.
```

```
  1  998.
  5  969.
  1 1033.
  1 1066.
  1 1088.
  1 1107.
  6 1184.
  1 1273.
  1 1309.
  1 1351.
  1 1374.
  1 1461.
  1 1494.
 10  246.
 15  742.
  7 1494.
@FIN
```

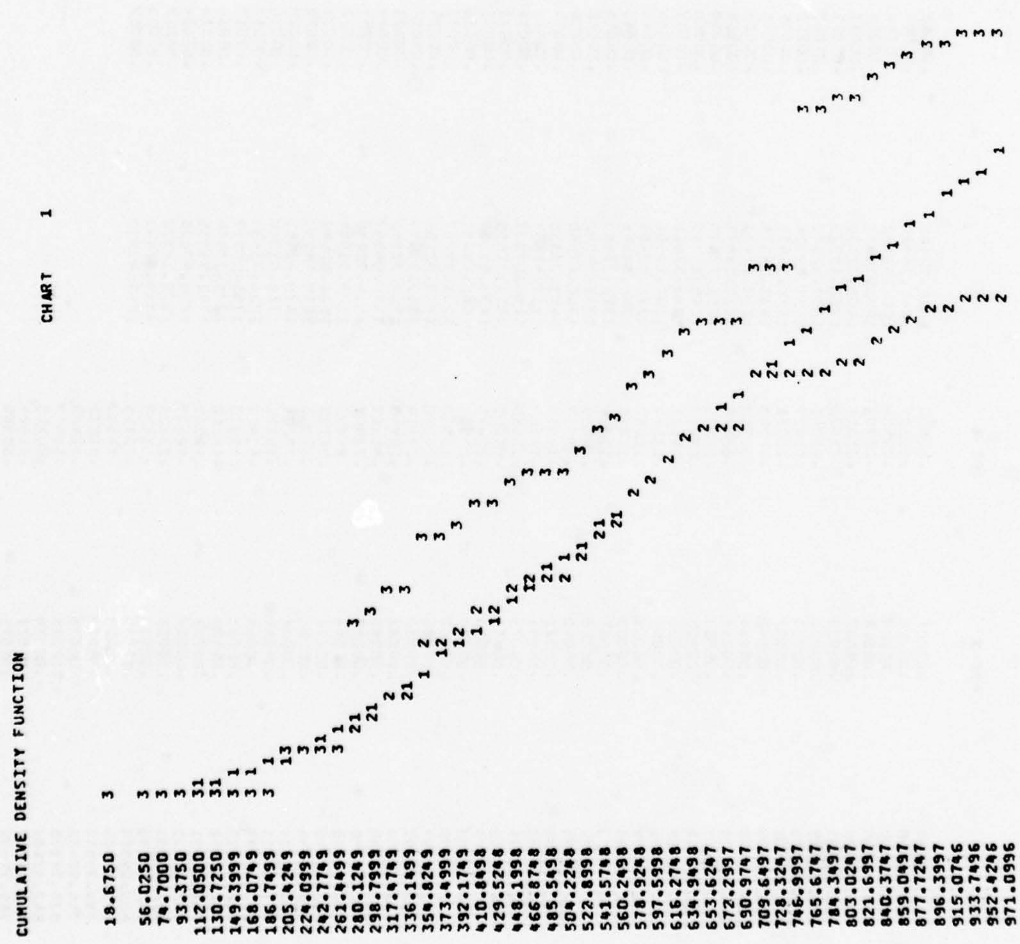
c. Program Output. The program output consists of the uncensored and censored data, moment and maximum likelihood parameter estimates, variance-covariance matrix, the pdf, the cdf and other information. Output from the above numerical example follows. Using the given inputs, the shape parameter (β) is calculated to be 1.91 and the scale parameter (α), 993.24. A shape parameter of 1.91 gives a positively skewed distribution lying between the exponential distribution ($\beta = 1$) and the normal distribution ($\beta = 3.5$). On the cdf graph, the ones (1) are the computed cdf, with the twos (2) representing the observed cdf using only the uncensored values, i.e., (Σ(uncensored value)/(total uncensored + total censored values). The threes (3) represent the observed cdf using all values, i.e., [Σ(uncensored values + censored values)]/(total uncensored + total censored).

Example

```
DATA FITTING OPTION HAS BEEN CHOSEN

NUMBER OF SAMPLES=     1
```

```
NO. COMPLETED =     68   NO. CENSORED =      32
  ARIT. MEAN          VARIANCE
   .7035+03           .3658+03

                                               OBSERVATIONS
 177.000   177.000   177.000   177.000   177.000   246.000   252.000   269.000   283.000   294.000
 331.000   331.000   331.000   331.000   331.000   367.000   379.000   386.000   411.000   423.000
 441.000   488.000   502.000   508.000   519.000   531.000   542.000   550.000   553.000   569.000
 583.000   589.000   601.000   613.000   621.000   682.000   682.000   682.000   682.000   692.000
 772.000   806.000   820.000   840.000   854.000   872.000   898.000   969.000   969.000   969.000
 969.000   969.000  1033.000  1066.000  1088.000  1107.000  1184.000  1184.000  1184.000  1184.000
1184.000  1184.000  1273.000  1309.000  1351.000  1374.000  1461.000  1494.000

                                        CENSORED OBSERVATIONS
 246.000   246.000   246.000   246.000   246.000   246.000   246.000   246.000   246.000   246.000
 742.000   742.000   742.000   742.000   742.000   742.000   742.000   742.000   742.000   742.000
 742.000   742.000   742.000   742.000   742.000  1494.000  1494.000  1494.000  1494.000  1494.000
1494.000  1494.000

MOMENT ESTIMATE OF THREE WEIBULL PARAMETERS FOR COMPLETED OBSERVATIONS
SCALE=    .9466758+03  SHAPE =    .2450397+01 LOCATION =     -.1360911+03

           COHEN*S VAR.-COV. MATRIX

         SHAPE                 SCALE


        .3376416-01          .1274180+06


        .1274180+06          .4852183+12


           ESSENWANGER*S VAR.-COV. MATRIX

         SHAPE                 SCALE


        .3376416-01          .5719916+09


        .5719916+09          .9689989+19


    EXP(X) - THEORETICAL MEAN =        881.16312          VARIANCE MEAN =        3105.96094
    VAR(X) =     229622.60937


         MAXIMUM LIKELIHOOD ESTIMATES
         SHAPE(BETA) =         1.9140920       SCALE(ALPHA) =       993.2397995
```

X	PDF X	CDF X
18.67500	.00005	.00050
37.35000	.00010	.00187
56.02500	.00014	.00406
74.70000	.00018	.00704
93.37500	.00022	.01077
112.05000	.00026	.01523
130.72500	.00030	.02041
149.40000	.00033	.02627
168.07500	.00037	.03281
186.74999	.00040	.03999
205.42499	.00043	.04780
224.09999	.00047	.05621
242.77499	.00050	.06521
261.44999	.00053	.07477
280.12499	.00055	.08486
298.79999	.00058	.09547
317.47499	.00061	.10657
336.14999	.00063	.11813
354.82499	.00065	.13014
373.49998	.00068	.14256
392.17498	.00070	.15537
410.84998	.00072	.16855
429.52498	.00073	.18207
448.19998	.00075	.19590
466.87498	.00076	.21002
485.54998	.00078	.22441
504.22498	.00079	.23904
522.89998	.00080	.25388
541.57497	.00081	.26890
560.24997	.00082	.28410
578.92496	.00082	.29943
597.59996	.00083	.31487
616.27496	.00083	.33041
634.94995	.00084	.34602
653.62495	.00084	.36168
672.29994	.00084	.37736
690.97494	.00084	.39304
709.64993	.00084	.40871
728.32493	.00084	.42433
746.99992	.00083	.43990
765.67492	.00083	.45540
784.34991	.00082	.47080
803.02491	.00082	.48609
821.69991	.00081	.50125
840.37490	.00080	.51627
859.04990	.00079	.53113
877.72489	.00078	.54582
896.39989	.00077	.56032
915.07488	.00076	.57463
933.74988	.00075	.58873
952.42487	.00074	.60260
971.09987	.00072	.61625
989.77486	.00071	.62966
1008.44986	.00070	.64282
1027.12485	.00068	.65573
1045.79984	.00067	.66837
1064.47482	.00066	.68075
1083.14981	.00064	.69285
1101.82480	.00063	.70468
1120.49979	.00061	.71622
1139.17477	.00060	.72748
1157.84976	.00058	.73845
1176.52475	.00056	.74914
1195.19974	.00055	.75953
1213.87473	.00053	.76964
1232.54971	.00052	.77945
1251.22470	.00050	.78897
1269.89969	.00049	.79821
1288.57468	.00047	.80716
1307.24966	.00046	.81582
1325.92465	.00044	.82420
1344.59964	.00043	.83230
1363.27463	.00041	.84012
1381.94962	.00040	.84767
1400.62460	.00038	.85495
1419.29959	.00037	.86197
1437.97458	.00035	.86872
1456.64957	.00034	.87522
1475.32455	.00033	.88147
1493.99954	.00031	.88747
1512.67453	.00030	.89323
1531.34952	.00029	.89876
1550.02451	.00028	.90406
1568.69949	.00027	.90914
1587.37448	.00025	.91399
1606.04947	.00024	.91864
1624.72446	.00023	.92308
1643.39944	.00022	.92732
1662.07443	.00021	.93137
1680.74942	.00020	.93523
1699.42441	.00019	.93891
1718.09940	.00018	.94242
1736.77438	.00017	.94576
1755.44937	.00017	.94893
1774.12436	.00016	.95194
1792.79935	.00015	.95481
1811.47433	.00014	.95753
1830.14932	.00013	.96011
1848.82431	.00013	.96255
1867.49930	.00012	.96486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93.3750
112.0500
130.7250
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168.0749
186.7499
205.4249
224.0999
242.7749
261.4499
280.1249
298.7999
317.4749
336.1499
354.8249
373.4999
392.1749
410.8498
429.5248
448.1998
466.8748
485.5498
504.2248
522.8998
541.5748
560.2498
578.9248
597.5998
616.2748
634.9498
653.6247
672.2997
690.9747
709.6497
728.3247
746.9997
765.6747
784.3497
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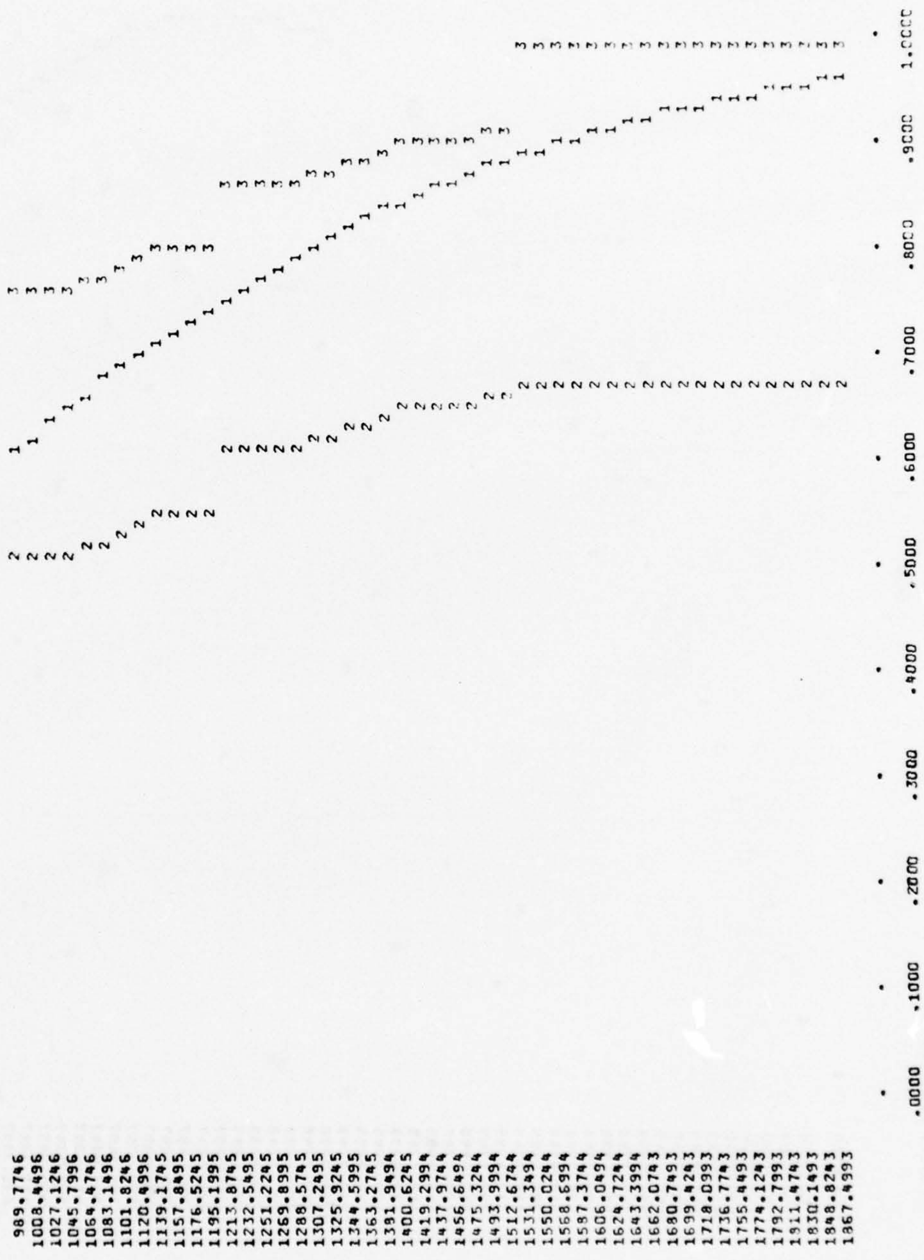
952.4246
971.0996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1493.9994
1512.6744
1531.3494
1550.0244
1568.6994
1587.3744
1606.0494
1624.7244
1643.3994
1662.0743
1680.7493
1699.4243
1718.0993
1736.7743
1755.4493
1774.1243
1792.7993
1811.4743
1830.1493
1848.8243
1867.4993


DENSITY FUNCTION
CHART 1
18.6750
56.0250
74.7000
93.3750
112.0500
130.7250
149.3999
168.0749
186.7499
205.4249
224.0999
242.7749
261.4499
280.1249
298.7999
317.4749
336.1499
354.8249
373.4999
392.1749
410.8498
429.5248
448.1998
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485.5498
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560.2498
578.9248
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653.6247
672.2997
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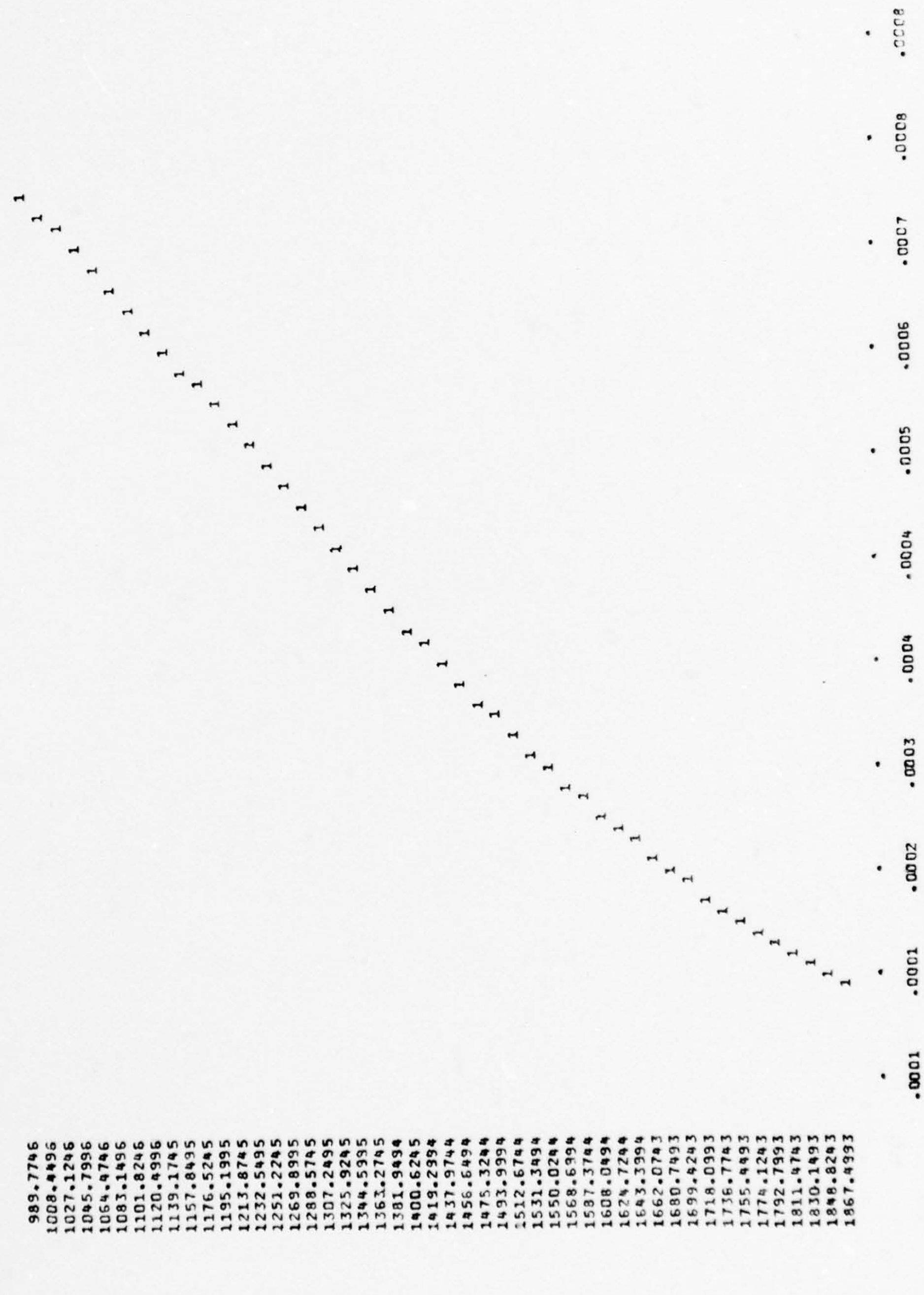
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1363.2745
1381.9494
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1456.6494
1475.3244
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1587.3744
1608.0494
1624.7244
1643.3994
1662.0743
1680.7493
1699.4243
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1848.8243
1867.4993
.0001
.0001
.0002
.0003
.0004
.0004
.0005
.0006
.0007
.0008
.0008
@FIN

APPENDIX A

STUDY CONTRIBUTORS

STUDY TEAM

a. Study Director

Mr. Jerry Thomas, Methodology, Resources, and Computation Directorate

b. Team Members

Cadet John R. F. Gallo, United States Military Academy
Mr. Keith D. Thorp, TRADOC Systems Analysis Activity

c. Support Personnel

Ms. Judy Bomstein, Graphics Branch
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Ms Joyce Garris, Word Processing Center
SFC R. D. Jones, Graphics Branch
Ms Thelma Laufer, Methodology, Resources and Computation Directorate
Ms Diane Passero, Word Processing Center

APPENDIX B

REFERENCES

1. Cohen, A. Clifford, "Maximum Likelihood Estimation in the Weibull Distribution Based On Complete and On Censored Samples," Technometrics, Vol. 7, No. 4, November 1965.

2. Essenwanger, Oskar M., "On Fitting of the Weibull Distribution with Non-Zero Location Parameter and Some Applications," Proceedings of the Thirteenth Conference on the Design of Experiments in Army Reserch Development and Testing, ARO-D Report 68-2, November 1968.

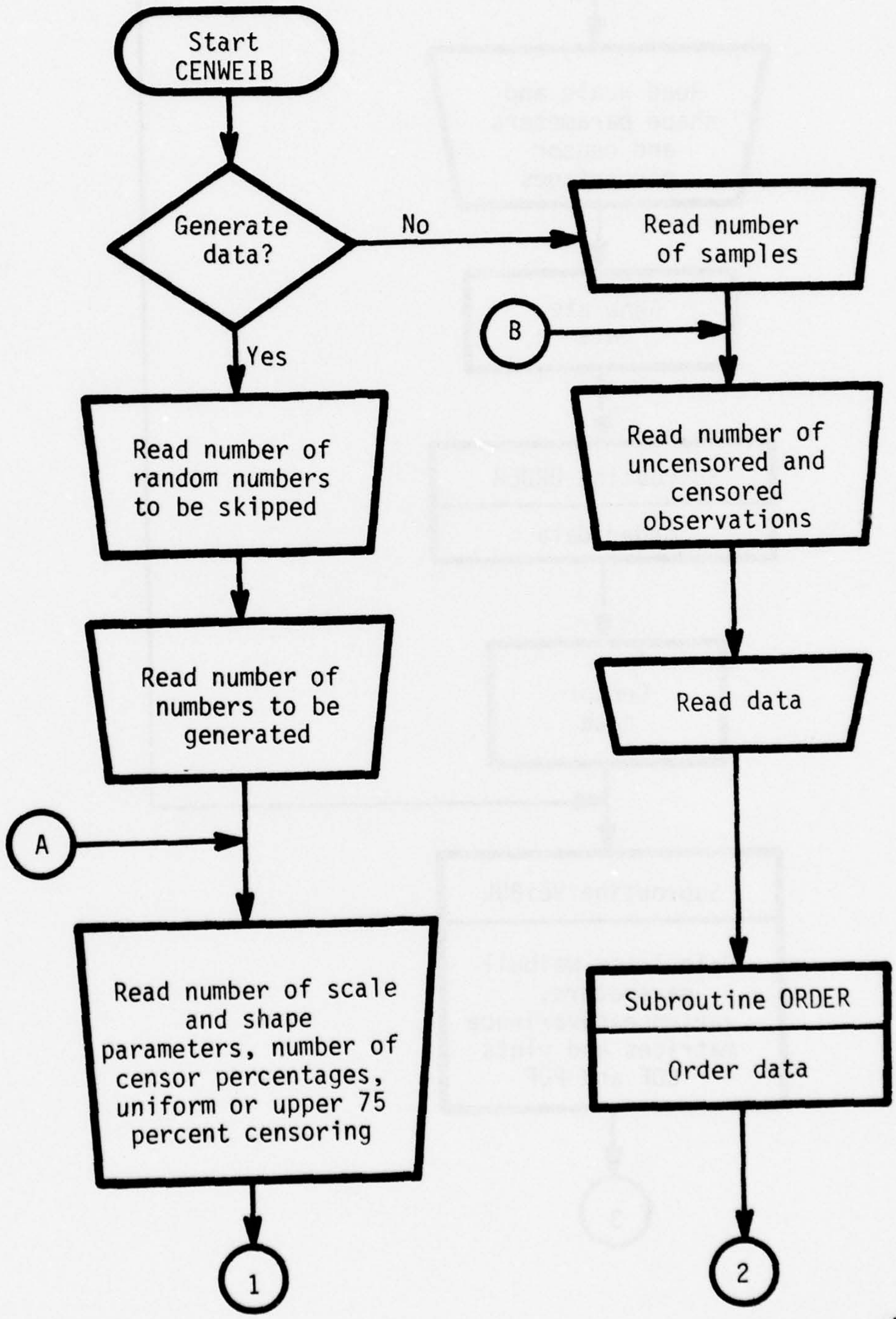
3. Kendall, M. G. and Alan Stuart, The Advanced Theory of Statistics, Vol. I, Second Edition, Hafner Publishing Co., New York, 1963, p. 356.

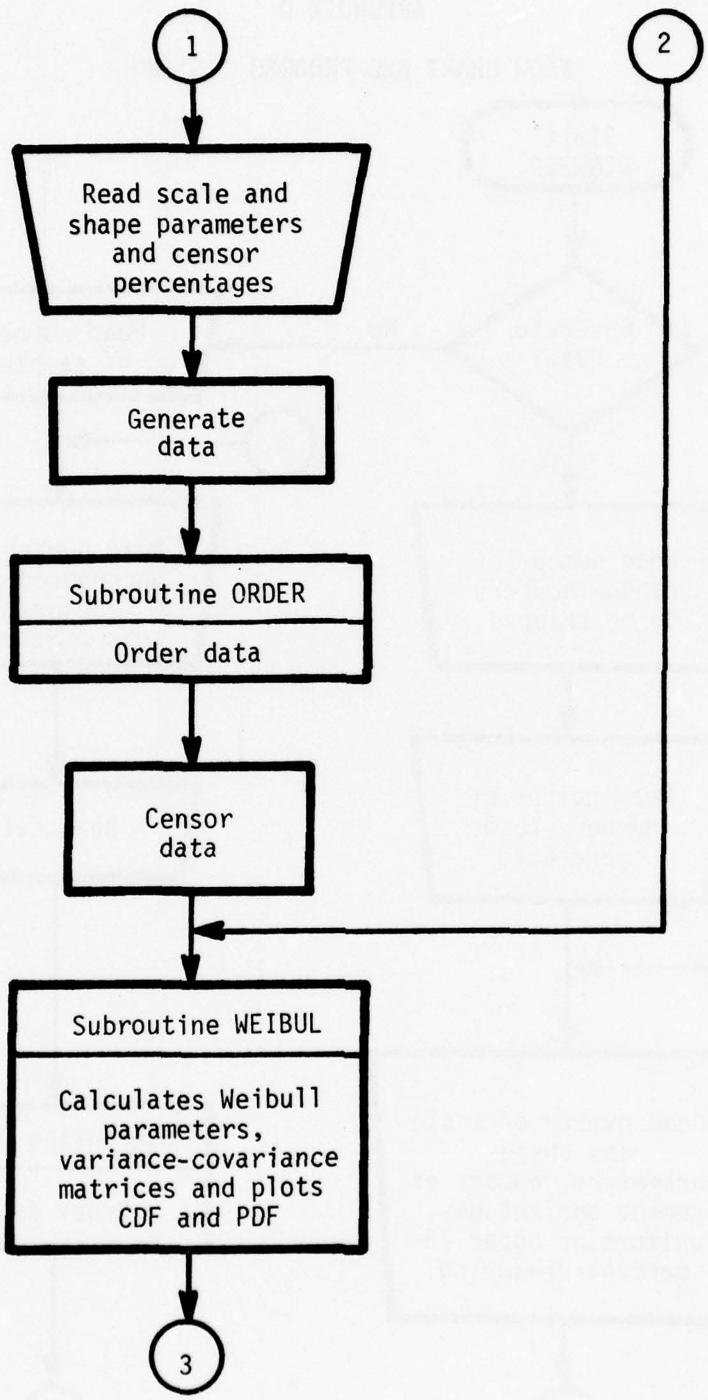
4. Bates, Carl B. and Jerry Thomas, "Application of Life Testing Techniques to Detection Data," CAA-TP-76-1, Technical Paper, March 1976.

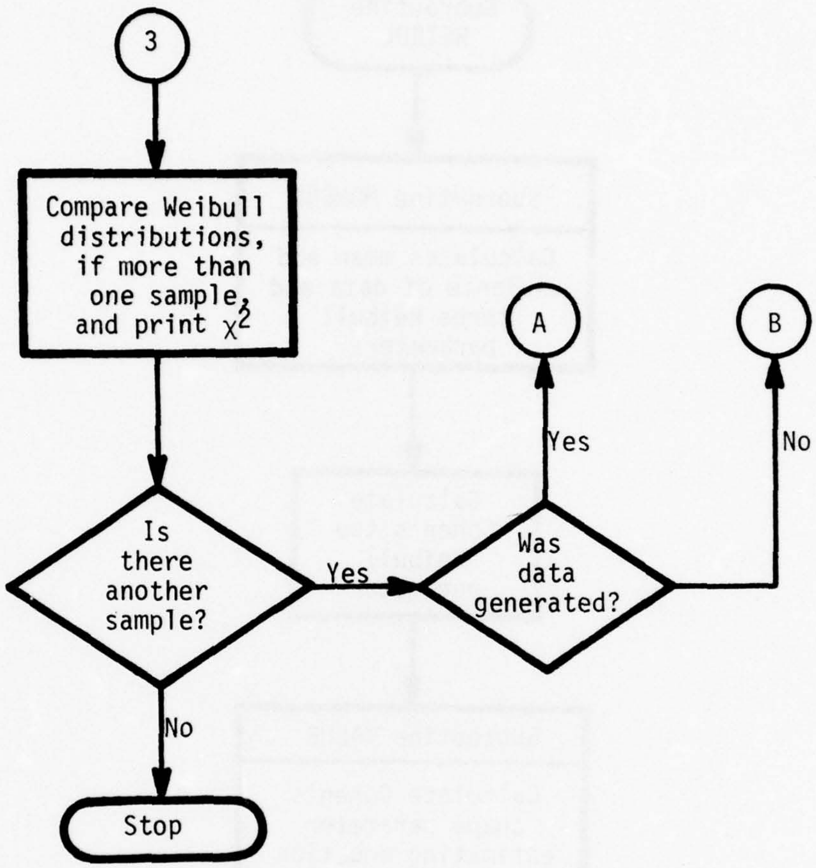
5. Thorp, Keith D., "Detection Process in Land Combat Field Experiments and Combat Models," unpublished paper.

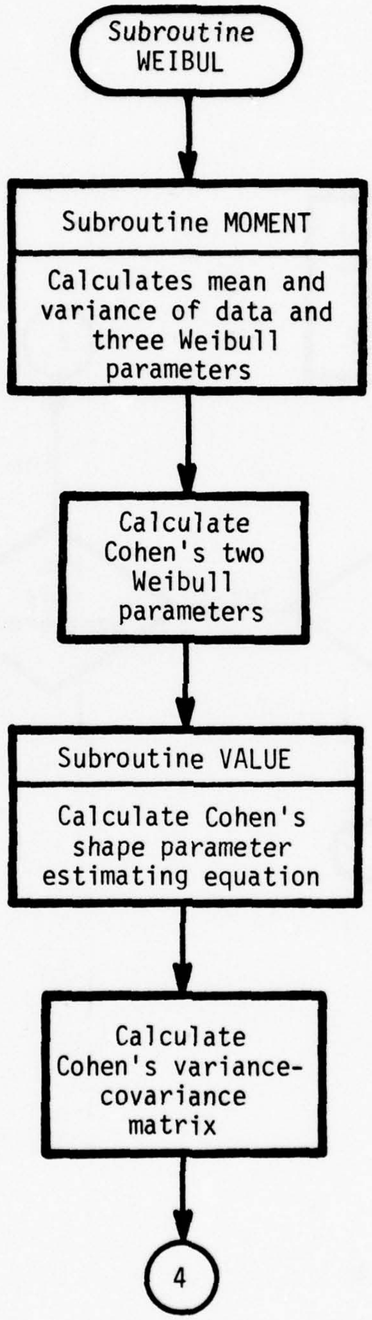
APPENDIX D

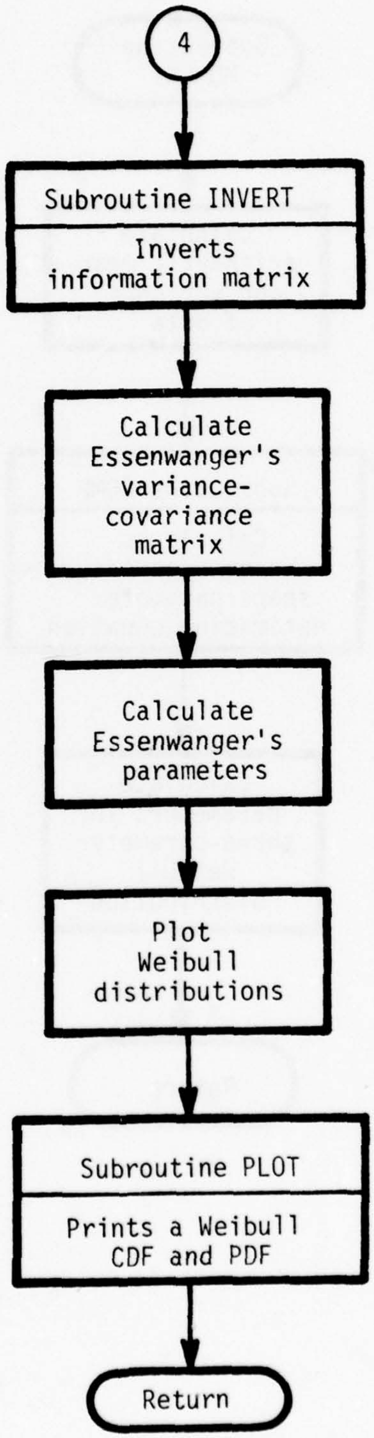
FLOW CHART AND PROGRAM LISTING

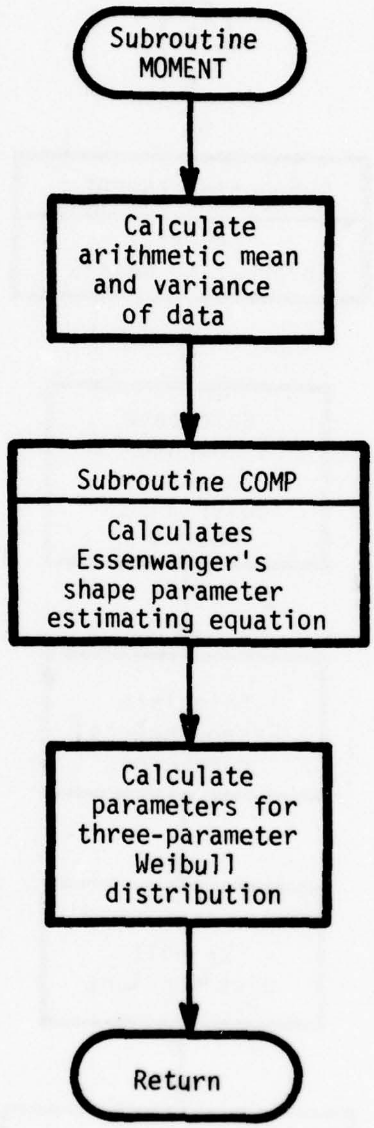



1
2
Read scale and shape parameters and censor percentages
Generate data
Subroutine ORDER
Order data
Censor data
Subroutine WEIBUL
Calculates Weibull parameters, variance-covariance matrices and plots CDF and PDF
3


3
Compare Weibull distributions, if more than one sample, and print χ^2
Is there another sample?
Yes
No
Stop
Was data generated?
Yes
No
A
B


Subroutine WEIBUL
Subroutine MOMENT
Calculates mean and variance of data and three Weibull parameters
Calculate Cohen's two Weibull parameters
Subroutine VALUE
Calculate Cohen's shape parameter estimating equation
Calculate Cohen's variance-covariance matrix
4


4
Subroutine INVERT
Inverts information matrix
Calculate Essenwanger's variance-covariance matrix
Calculate Essenwanger's parameters
Plot Weibull distributions
Subroutine PLOT
Prints a Weibull CDF and PDF
Return


Subroutine MOMENT
Calculate arithmetic mean and variance of data
Subroutine COMP
Calculates Essenwanger's shape parameter estimating equation
Calculate parameters for three-parameter Weibull distribution
Return

```
C*******************************************************************************
C-------------------------------------------------------------------------------
C       PROGRAM NAME - CENWEIB
C-------------------------------------------------------------------------------
C       CENWEIB GENERATES 500 WEIBULLY DISTRIBUTED NUMBERS WITH PRE-SPECIFIED
C       ALPHA(SCALE) AND BETA(SHAPE) PARAMETERS AND RANDOMLY CENSORS FROM THE
C       UPPER SEVENTY-FIFTH PERCENTILE OF THE 500 NUMBERS OR FITS A WEIBULL
C       CURVE TO INPUT DATA FROM COMPLETE, SINGLY-CENSORED, OR PROGRESSIVELY-
C       CENSORED SAMPLES.
C-------------------------------------------------------------------------------
C       INPUT-------------------------------------------------------------------
C       CARD 1
C       0 TO FIT CURVE TO GIVEN DATA IN FIVE COLUMN FIELDS.
C       ANY OTHER NUMBER TO GENERATE DATA.
C       CARD 2
C       IF CARD 1 WAS 0, NUMBER OF SAMPLES IN FIRST FIVE COLUMN FIELDS (COL 1-5).
C       OTHERWISE, NUMBER OF RANDOM NUMBERS TO BE DISCARDED.
C       CARD 3
C       IF CARD 1 WAS 0, NUMBER OF UNCENSORED DATA GROUPS, NUMBER OF CENSORED DATA
C       GROUPS IN FIVE COLUMN FIELDS.
C       OTHERWISE, NUMBER OF WEIBULL NUMBERS TO BE GENERATED IN FIVE COLUMN
C       FIELDS.
C       CARD 4
C       IF CARD 1 WAS 0, FREQUENCY OF OBSERVATIONS (IF LEFT BLANK, IT IS SET
C       EQUAL TO 1) IN FIVE COLUMN FIELDS, VALUE OF DATA GROUP IN A TEN
C       COLUMN FIELD.
C       OTHERWISE, NUMBER OF ALPHAS, NUMBER OF BETAS, NUMBER OF DIFFERENT
C       PERCENTAGES FOR CENSORING, AND A 0 FOR UNIFORM CENSORING OR 1 FOR
C       CENSORING UPPER 75 PERCENT IN FIVE COLUMN FIELDS.
C       *NEXT THREE CARDS ARE USED ONLY IF DATA IS TO BE GENERATED.
C       CARD 5
C       THE ALPHA VALUE(S) IN TEN COLUMN FIELDS.  MAXIMUM OF SIX VALUES.
C       CARD 6
C       THE BETA VALUE(S) IN TEN COLUMN FIELDS.  MAXIMUM OF 25 VALUES.
```

```
C     CARD 7
C     THE PERCENTAGE(S) CENSORED (WITH DECIMAL PUNCHED) IN TEN COLUMN FIELDS.
C     MAXIMUM OF SIX VALUES.
C-----------------------------------------------------------------------
C     OUTPUT----------------------------------------------------------------
C     1 - IF CARD 1 IS 0, NUMBER OF SAMPLES.
C         IF CARD 1 IS ANY OTHER NUMBER, NUMBER OF WEIBULL NUMBERS GENERATED.
C     2 - STATEMENT OF UNIFORM CENSORING OR CENSORING OVER UPPER 75 PERCENT
C         (CARD 1.NE.0, ONLY).
C     3 - NUMBER OF COMPLETED AND CENSORED OBSERVATIONS.
C     4 - MEAN AND VARIANCE OF OBSERVATIONS.
C     5 - CALCULATED PARAMETER VALUES.
C     6 - THE DATA (INPUT OR GENERATED).
C     7 - MOMENT ESTIMATE OF THREE WEIBULL PARAMETERS FOR COMPLETED
C         OBSERVATIONS.
C     8 - COHEN*S AND ESSENWANGER*S VARIANCE-COVARIANCE MATRIX.
C     9 - THEORETICAL MEAN FOR TRANSFORMED AND UNTRANSFORMED OBSERVATIONS,
C         VARIANCE MEAN.
C    10 - MAXIMUM LIKELIHOOD ESTIMATES OF THE PARAMETER VALUES.
C    11 - A LISTING OF 100 VALUES FOR THE INDEPENDENT VARIABLE X, PDF X,
C         CDF X.
C    12 - A PLOT OF THE CDF WITH UPPER AND LOWER LIMITS.
C         1 - DENOTES THE OBSERVED CDF.
C         2 - LOWER LIMIT (DISCOUNTING CENSORED OBSERVATIONS).
C         3 - UPPER LIMIT (ASSUMING NO CENSORED OBSERVATIONS).
C    13 - A PLOT OF THE PDF X.
C    14 - A CHI-SQUARE STATISTIC COMPARING PAIRS OF DISTRIBUTIONS.
C    15 - LISTING OF PERCENTAGES OF CENSORING, INPUT ALPHAS, ALPHA HATS,
C         BETAS, BETA HATS, THEORETICAL MEANS OF THE OBSERVATIONS,
C         VARIATIONS OF THE OBSERVATIONS, VARIATIONS OF THE MEANS
C         (FOR CARD 1.NE.0 ONLY).
      READ(5,286)IDATA
  286 FORMAT(I5)
```

```
      DIMENSION X(1000)
      DIMENSION C(2,2,50),P(2,50),D(2,2),E(2),F(2)
      COMMON /BATES/KOUNT
      KOUNT=0
      IF(IDATA.EQ.0)GO TO 1000
C
C   GENERATE RANDOM DATA FROM WEIBULL DISTRIBUTION
C
      WRITE(6,80)
   80 FORMAT(1H1,46HWEIBULL DATA GENERATION OPTION HAS BEEN CHOSEN)
      DIMENSION W(1000)
      REAL ME
      DIMENSION ALPHA(6),BETA(25),CEN(6)
      DIMENSION VAX(200),VAXH(200),B1(200),B2(200),B1B2(200)
      DIMENSION I1(200),K1(200),CHI(200)
      DIMENSION PCEN(200),AL(200),ALH(200),BE(200),BEH(200),ME(200)
      COMMON PCEN,AL,ALH,BE,BEH,ME,VAX,VAXH
      READ(5,288)IZZQ
      READ(5,288)INO
  288 FORMAT(I5)
      WRITE(6,21)INO
   21 FORMAT(1H0,41HNUMBER OF RANDOM NUMBERS TO BE GENERATED=, I5)
  350 READ(5,67,END=200) NA,NB,NC,NS
      LL=1
   67 FORMAT(4I5)
       WRITE(6,290)NS
  290 FORMAT(1H0,5HNS = ,I5,17H :    0-UNIFORMLY ,
     *37HCENSORED    1-CENSORED OVER UPPER 75%   )
      READ(5,68)  (ALPHA(I),I=1,NA)
      READ(5,68) (BETA(I),I=1,NB)
      READ(5,68) (CEN(I),I=1,NC)
       NRSAMP=NA*NB*NC
```

```
C
C     NRSAMP IS NUMBER OF DATA SAMPLES BEING GENERATED
C
      WRITE(6,22)
   22 FORMAT(1H0,9HALPHA(S)=)
      PRINT 68,(ALPHA(I),J=1,NA)
      WRITE(6,23)
   23 FORMAT(1H0,8HBETA(S)=)
      PRINT 68,( BETA(I),I=1,NB)
      WRITE(6,24)
   24 FORMAT(1H0,21HCENSOR PERCENTAGE(S)=)
      PRINT 68 ,( CEN(I),I=1,NC)
  68  FORMAT(6F10.2)
      L1=1
   42  L2=1
   41  L3=1
   40 GAMMA=0
      SR=0.
      SR2=0.
      DO 66 I=1,IZZQ
   66 R = BARN(+1)
  69  DO 7 I=1,INO
      R =BARN(+1)
      X(I)=GAMMA+ALPHA(L1)*(-ALOG(R))**(1./BETA(L2))
  7   CONTINUE
      CALL ORDER(INO,X)
      K=INO *CEN(L3)+.5
      N=INO-K
      M=N
      IF(K.EQ.0) GO TO 45
        IKOUNT=INO
C
C    RANDOMLY CENSORS GENERATED VALUES
C
```

```
      DO 30 I=1,K
 35   IKOUNT=IKOUNT+1
      R =BARN(+1)
      NN=R*INC
      XPOINT=  0.
        IF(NS.EQ.0)   GO TO 7375
C
C   CENSORS UPPER 75 PERCENT
C
        JJ1=INO*.25
      XPOINT=   X(JJ1)
      IF(NN.LE.JJ1) GO TO 35
 7375 CONTINUE
C
C   UNIFORMLY CENSOR
C
      IKOUNT=IKOUNT+1
      R =BARN(+1)
      IF(X(NN).EQ.0.0) GO TO 35
      CN     =(X(NN)-XPOINT)*R+XPOINT
      M=M+1
      W(M)=CN
      X(NN)=0.0
 30   CONTINUE
  45  II=0
      DO 50 I=1,INO
      IF(X(I).EQ.0.0) GO TO 50
      II=II+1
      W(II)=X(I)
 50   CONTINUE
      ALPHH=ALPHA(L1)
      BETT=BETA(L2)
      CALL WEIBUL(W,N,K,ALPHH,BETT,GAMMA,C,D,E,F,P)
```

```
      PCEN(KOUNT) = CEN(L3)
      AL(KOUNT) = ALPHA(L1)
      BE(KOUNT) = BETA(L2)
      IF(KOUNT.EQ.1) GO TO 1011
      L=KOUNT-1
      WRITE(6,51)
   51 FORMAT(1H1)
      GO TO 747
 1000 READ 1002,NRSAMP
 1002 FORMAT(16I5)
C
C   FIT INPUT DATA TO WEIBULL DISTRIBUTICN
C
      WRITE(6,81)
   81 FORMAT(1H1,35HDATA FITTING OPTION HAS BEEN CHOSEN)
      WRITE(6,20)NRSAMP
   20 FORMAT(1H0,18HNUMBER OF SAMPLES=, I5)
      K=0
    1 READ 1002,N,K
      IC=0
      IK=0
      IN=0
C  N T IS NOW THE NUMBER OF CARDS TO BE READ
      NT= K+N
        DO 2 I=1,NT
      READ 1003,JFREQ,VALUE
      IF(JFREQ.LE.0) JFREQ=1
 1003 FORMAT(I5,F10.5)
      J=JFREQ
      DO 3 L=1,J
      IF(I.LE.N)  IN=IN+1
      IF(I.GT.N)    IK=IK+1
      IC=IC+1
```

```
    3 X(IC)=VALUE
    2 CONTINUE
      K=IK
      N=IN
      NT=IC
      CALL WEIBUL(X,N,K,ALPHH,BETT,GAMMA,C,D,E,F,P)
      IF(NRSAMP.EQ.1)GO TO 433
      IF(KOUNT.EQ.1) GO TO 1
      L=KOUNT-1
  747  I=1
C
C   USE CHI-SQUARE STATISTIC WITH TWO DEGREES OF FREEDOM TO TEST
C   STATISTICAL SIMILARITY OF TWO OR MORE SAMPLES
C
   54 DO  341 J=1,2
      DO  341 K=1,2
  341 D(J,K)= C(J,K,I)+C(J,K,KOUNT)
      CALL INVERT(D,2,E,DET)
      IF(DET.EQ.0.) PRINT 1040,DET
1040  FORMAT(22H MATRIX DID NOT INVERT,F10.0)
      DO 343 J=1,2
  343 E(J)=P(J,I)-P(J,KOUNT)
      DO  342 K=1,2
      F(K)=0.
      DO  342 J=1,2
  342 F(K)=D(K,J)*E(J) +F(K)
      CHISQ = F(1)*E(1) +F(2)*E(2)
      PRINT 1050, I,KOUNT,CHISQ
1050  FORMAT(//10X,7HSAMPLE ,I5,13H WITH SAMPLE ,I5,10H  CHISQ = ,E15.7,
     129H  WITH TWO DEGREES OF FREEDOM)
  433 IF(IDATA.EQ.0)GO TO 643
      I1(LL) = I
      K1(LL) = KOUNT
```

```
      CHI(LL) = CHISQ
      B1(LL) = BEH(I)
      B2(LL) = BEH(KOUNT)
      B1B2(LL) = B2(LL) -B1(LL)
       LL= LL + 1
  643 I=I+1
      IF(I.LE.L)GO TO 54
      IF(IDATA.EQ.0)GO TO 53
      GO TO 1011
   53 IF(KCUNT.GE.NRSAMP)GO TO 203
      GO TO 1
 1011 L3=L3+1
      IF(L3.LE.NC)GO TO 40
      L2=L2+1
      IF(L2.LE.NB)GO TO 41
      L1=L1+1
      IF(L1.LE.NA)GO TO 42
      GO TO 350
  200 LL = LL - 1
      WRITE(6,39)
   39 FORMAT(1H0,45H  P CEN    ALPHA  ALPHA HAT      BETA  BETA HAT     ,
     130H      E(X)     VAR(X)  VAR(XBAR))
      DO 450 I = 1,KOUNT
  450 PRINT 38, PCEN(I),AL(I),ALH(I),BE(I),BEH(I),ME(I),VAX(I),VAXH(I)
   38 FORMAT(2F8.4,1F11.4,1F9.4,1F10.4,1F9.4,1F10.4,1F11.4)
      IF(NRSAMP.EQ.1) GO TO 203
      WRITE(6,489)
  489 FORMAT(1H0,41HSAMPLE WITH SAMPLE  BETA HAT 1 BETA HAT 2    ,
     122H  DIFFERENCE      CHISQ)
       DO 490 I = 1,LL
  490 PRINT 494, I1(I),KI(I),B1(I),B2(I),B1B2(I),CHI(I)
  494 FORMAT(I5,7X,I5,1F14.5,1F11.5,1F12.5,1F10.5)
  203 STOP
      END
```

```
      SUBROUTINE ORDER(N,X)
C
C  THIS SUPROUTINE ARRANGES INPUT DATA IN INCREASING OPDER.
C
      DIMENSION X(1)
      DO 4 I=1,N
      DO 4 J=I,N
      IF(X(I).GT.X(J)) GO TO 5
      GO TO 4
5     T=X(I)
      X(I)=X(J)
      X(J)=T
4     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE COMP(A1,B,FN,A,B1)
C
C  THIS SUBROUTINE CALCULATES ESSENWANGER'S SHAPE PARAMETER MOMENT
C  ESTIMATING EQUATION.
C
      CALL GAMMA((1+1/B),A,$1,$150)
      GO TO 2
    1 A=ALOG(A)
    2 CALL GAMMA((1+2/B),B1,$4,$150)
      GO TO 3
    4 B1=10.**B1
    3 CALL GAMMA((1+3/B),C,$5,$150)
      GO TO 6
    5 C=ALOG(C)
    6 CONTINUE
      ANUM= C-3.*A*B1+2.*A**3
      DENOM= (B1-A*A)**1.5
      FN= ANUM/DENOM -A1
      GO TO 160
  150 WRITE(6,151)
  151 FORMAT(1H ,10X,28HGAMMA VALUE NEGATIVE OR ZERO)
  160 RETURN
      END
```

```
      SUBROUTINE VALUE(X,N,NT,B,FUN)
C
C THIS SUBROUTINE CALCULATES COHEN'S SHAPE PARAMETER LIKELIHOOD
C ESTIMATING EQUATION.
C
      DIMENSION X(1)
      SLNX=0.
C     SUM ALOG(X(I))
      DO 20 I=1,N
   20 SLNX=SLNX + ALOG(X(I))
      SLNX=SLNX/N
      TOP=0.
      DO 30 I=1,NT
   30 TOP=TOP+ ALOG(X(I))*X(I)**B
      DENOM=0.
      DO 40 I=1,NT
   40 DENOM= DENOM + X(I)**B
      FUN= TOP/DENOM -1./B -SLNX
      RETURN
      END
```

```
      SUBROUTINE MOMENT(X,N,ALPHA,GAM,BETA)
C
C  THIS SUBROUTINE CALCULATES THE ARITHMETIC MEAN AND VARIANCE OF THE
C  INPUT DATA.  IT ALSO CALCULATES THE PARAMETERS OF A
C  THREE-PARAMETER WEIBULL DISTRIBUTION FOR THE DATA USING
C  ESSENWANGER'S MOMENT-ESTIMATING TECHNIQUE.
C
      DIMENSION X(1)
C
C  CALCULATES ARITHMETIC MEAN AND VARIANCE OF DATA.
C
      SR=0.
      DO 304 I=1,N
  304 SR=SR+X(I)
      AMEAN= SR/N
      SR= 0.
      SR3=0.
      DO 305 I=1,N
      SR=SR+(X(I)-AMEAN)**2
  305 SR3=SR3+(X(I)-AMEAN)**3
      SIG= SQRT(SR/N)
      E3= SR3/N
      A1=E3/SIG**3
      WRITE(6,202)
  202 FORMAT(5X,25HARIT. MEAN        VARIANCE)
      PRINT 205 ,AMEAN,SIG
C
C  CALCULATES THREE WEIBULL PARAMETERS.
C
      FN=1.
      B=-.4
      BINV=.5
      DO 210 J=1,4
  201 BL=B
```

```
      FNL=FN
      B=B+BINV
      CALL COMP(A1,B,FN,A,B1)
      IF(FN.GT.0.) GO TO 201
      BINV=BINV/10.
      B=BL+(B-BL)*ABS(FNL)/(ABS(FN)+ABS(FNL))
      IF(J.EQ.4) GO TO 210
  206 B=B-BINV
      CALL COMP(A1,B,FN,A,B1)
      IF(FN.LT.0.) GO TO 206
  210 CONTINUE
      CALL COMP(A1,B,FN,A,B1)
      DENOM=B1-A*A
      ALPHA= SQRT(SIG*SIG/DENOM    )
      GAM=AMEAN-ALPHA*A
      BETA = B
  205 FORMAT(5E15.4)
      RETURN
  308 CONTINUE
      END
```

```
      SUBROUTINE INVERT (A, N, INDEX, DET)
C     THIS SUBROUTINE PERFORMS A DOUBLE PRECISION INVERSION OF A MATRIX.
C     WHILE THIS ROUTINE MAY BE USED FOR ANY MATRIX WHICH DOES NOT HAVE
C     ELEMENT ON THE MAIN DIAGONAL EQUAL TO ZERO, IT IS DESIGNED PRI-
C     MARILY FOR POSITIVE DEFINITE MATRICES.
C
C     INPUT--
C     1)  A(N,N) IS THE MATRIX TO BE INVERTED WHICH WILL BE REPLACED BY
C     THE INVERSE.  ALTHOUGH THE INVERSION IS IN DOUBLE PRECISION, THE
C     MATRIX IS INPUT AND ITS INVERSE IS OUTPUT IN SINGLE PRECISION.
C     2)  N IS THE ORDER OF THE MATRIX.
C     3)  INDEX(N) IS A TEMPORARY STORAGE VECTOR USED BY THE SUBROUTINE.
C     IT IS INCLUDED IN THE CALL LIST TO AVOID PLACING A LIMIT ON THE
C     DIMENSION OF A.
C
C     OUTPUT--
C     ZERO.
      DIMENSION A(N,N), INDEX(N)
C
      DET = 1.0
      DO 10  I = 1, N
   10 INDEX(I) = 0
      DO 100  I = 1, N
C
C     FIND LARGEST DIAGONAL ELEMENT.
   20 AMAX = 0.
      DO 50  J = 1, N
      IF (INDEX(J) - 1)  30, 50, 30
   30 IF(AMAX - A(J,J))  40, 50, 50
   40 IP = J
      AMAX = A(J,J)
   50 CONTINUE
```

```
            INDEX(IP) = 1
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT.
            P = A(IP,IP)
            DET = DET*P
            A(IP,IP) = 1.
            DO 60  L = 1, N
            A(IP,L) = A(IP,L)/P
       60   CONTINUE
C
C     REDUCE NON-PIVOT ROWS.
            DO 90  L1 = 1, N
            IF (L1 - IP)  70, 90, 70
       70   T = A(L1,IP)
            A(L1,IP) = 0.
            DO 80  L = 1, N
            D = A(L1,L)
            E = A(IP,L)
            D = D - E*T
       80   A(L1,L) = D
      90    CONTINUE
      100   CONTINUE
      110   RETURN
C
C     SINGULAR MATRIX RETURN
      END
```

```
      SUBROUTINE PLOT(NO,A,N,M,NL,NS)
C
C  THIS SUBROUTINE PRINTS A WEIBULL CUMULATIVE DISTRIBUTION
C  FUNCTION AND A WEIBULL PROBABILITY DISTRIBUTION FUNCTION.
C
      DIMENSION OUT(101),YPR(11),ANG(9),A(1)
      DATA BLANK/4H    /,ANG/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9/
    1 FORMAT(1H0,60X,7H CHART ,I5,//)
    2 FORMAT(1H ,F11.4,5X,101A1)
    3 FORMAT(1H  )
    4 FORMAT(10H 123456789)
    5 FORMAT(10A1)
    7 FORMAT(1H0,16X,101H.          .          .          .          .
     1    .          .          .          .          .          .)
    8 FORMAT(1H0,9X,11F10.4)
C
      NLL=NL
C
      IF(NS) 16,16,10
C
C        SORT BASE VARIABLE DATA IN ASCENDING ORDER
C
   10 DO 15 I=1,N
      DO 14 J=I,N
      IF(A(I)-A(J)) 14,14,11
   11 L=I-N
      LL=J-N
      DO 12 K=1,M
      L=L+N
      LL=LL+N
      F=A(L)
      A(L)=A(LL)
   12 A(LL)=F
```

```
     14 CONTINUE
     15 CONTINUE
C
C          TEST NLL
C
     16 IF(NLL) 20,18,20
     18 NLL= 50
C
C        PRINT TITLE
C
     20 PRINT 1, NO
C
C         DEVELOP BLANK AND DIGITS FOR PRINTING
C
C       REWIND 13
C       WRITE(13,4)
C       REWIND 13
C        READ(13,5) BLANK,( ANG(I),I=1,9)
C       REWIND 13
C
C        FIND SCALE FOR BASE VARIABLE
C
        XSCAL=(A(N)-A(1))/(FLOAT(NLL-1))
C
C          FIND SCALE FOR CROSS-VARIABLES
C
        M1=N+1
        YMIN=A(M1)
        YMAX=YMIN
        M2=M*N
        DO 40 J=M1,M2
        IF(A(J)-YMIN)  28,26,26
     26 IF(A(J)-YMAX) 40,40,30
```

```
   28 YMIN=A(J)
      GO TO 40
   30 YMAX=A(J)
   40 CONTINUE
      YSCAL=(YMAX-YMIN)/100.0
C
C             FIND BASE VARIABLE PRINT POSITION
      XB=A(1)
      L=1
      MY=M-1
      I=1
   45 F=I-1
      XPR=XB+F*XSCAL
      IF(A(L)-XPR) 50,50,70
C
C         FIND CROSS-VARIABLES
C
   50 DO 55 IX=1,101
   55 OUT(IX)=BLANK
      DO 60 J=1,MY
      LL=L+J*N
      JP=((A(LL)-YMIN)/YSCAL)+1.0
      OUT(JP)=ANG(J)
   60 CONTINUE
C
C         PRINT LINE AND CLEAR, OR SKIP
C
      PRINT 2,    XPR,(OUT(IZ),IZ=1,101)
      L=L+1
      GO TO 80
   70 PRINT 3
   80 I=I+1
      IF(I-NLL) 45,84,86
```

```
      84 XPR=A(N)
         GO TO 50
C           PRINT CROSS-VARIABLES NUMBERS
C
      86 PRINT 7
         YPR(1)=YMIN
         DO 90 KN=1,9
      90 YPR(KN+1)=YPR(KN)+YSCAL*10.0
         YPR(11)=YMAX
         PRINT 8,   (YPR(IP),IP=1,11)
         RETURN
         END
```

```
      SUBROUTINE WEIBUL(X,N,K,A,B,G,C,D,E,F,P)
C
C  THIS SUBROUTINE CALCULATES:
C        1 - PARAMETERS FOR THE THREE-PARAMETER WEIBULL DISTRIBUTION
C        2 - COHEN'S VARIANCE-COVARIANCE MATRIX
C        3 - ESSENWANGER'S VARIANCE-COVARIANCE MATRIX
C        4 - ESSENWANGER'S PARAMETERS FOR THE TWO-PARAMETER WEIBULL
C             DISTRIBUTION
C  AND PLOTS THE TWO-PARAMETER CDF AND PDF.
C
      REAL ME
      DIMENSION C(2,2,50),P(2,50),D(2,2),E(2),F(2)
      DIMENSION X(1000),Z(100,5)
      DIMENSION PCEN(200),AL(200),ALH(200),BE(200),BEH(200),ME(200)
      DIMENSION VAX(200),VAXH(200)
      COMMON PCEN,AL,ALH,BE,BEH,ME,VAX,VAXH
C     REAL DB,DDB,DA,DDA,ONE,DMEAN
      DIMENSION ACV(2,2),AP(2,2),APT(2,2)
      COMMON /BATES/KOUNT
C   1 READ 1002,N,K
      PRINT 976,N,K
976   FORMAT(1H1,18H  NO. COMPLETED = ,I5,20H   NO. CENSORED =    ,I5)
      NT=N+K
      L=N+1
      CALL ORDER(N,X)
      IF(K.EQ.0) GO TO 900
      CALL ORDER(K,X(N+1))
  900 CONTINUE
C  TEST FOR ZERO VALUES
      DO 2 I=1,NT
      IF(X(I).EQ.0.) GO TO 3
    2 CONTINUE
      GO TO 4
    3 DO 5 I=1,NT
```

```
    5 X(I)=X(I) +.001
C
C  PARAMETERS FOR THREE-PARAMETER WEIBULL DISTRIBUTION CALCULATED.
C
    4 CALL MOMENT(X,N,A,G,B)
      PRINT 1004,(X(I),I=1,N)
 1004 FORMAT(//50X,12HOBSERVATIONS/ (10F10.3))
      IF(K.EQ.0) GO TO 1010
      PRINT 1005,(X(I),I=L,NT)
1005  FORMAT(//40X,21HCENSORED OBSERVATIONS/(10F10.3))
      GO TO 1011
 1010 PRINT 1007
1007  FORMAT(//40X,26H NO CENSORED OBSERVATIONS ///)
1006  FORMAT(/,71H MOMENT ESTIMATE OF THREE WEIBULL PARAMETERS FOR COMPL
     1ETED OBSERVATIONS/7H SCALE=,E15.7,9H  SHAPE =,E15.7,12H LOCATION =
     2 ,E15.7)
 1011 PRINT 1006,A,B,G
C
C  COHEN'S PARAMETERS AND VARIANCE-COVARIANCE MATRIX ARE CALCULATED.
C
      G=0.
      DISC=.001
C    START LOOP TO DETERMINE ROOT OF FUNTION GIVEN BY VALUE.
C    B IS THE PARAMETER
      CALL VALUE(X,N,NT,B,FN)
      BINV=100*DISC
      DO 210 J=1,5
  201 BL=B
      FNL=FN
      B=B+BINV
      CALL VALUE(X,N,NT,B,FN)
 1009 FORMAT(5E15.7)
      IF(SIGN(1.,FNL).NE.SIGN(1.,FN)) GO TO 210
      IF(ABS(FNL).LT.ABS(FN)) BINV=-BINV
```

```
      GO TO 201
  210 BINV=BINV/10.
      B= BL+(B-BL)*ABS(FNL)/(ABS(FN)+ABS(FNL))
      CALL VALUE(X,N,NT,B,FN)
      SUM=0.
      DO 300  I=1,NT
  300 SUM= SUM + (X(I)**B)/N
      A= SUM
      KOUNT=KOUNT+1
 COMPUTE SIGMA**2 B
      SUM=0.
      DO 310 I=1,NT
  310 SUM= SUM + (X(I)**B)*(ALOG(X(I))**2)
      C(1,1,KOUNT)=  N/B**2 + SUM/A
      SUM=0.
      DO 320 I=1,NT
  320 SUM=SUM +(X(I)**B)*ALOG(X(I))
      C(1,2,KOUNT)= - SUM/A**2
      C(2,1,KOUNT)= C(1,2,KOUNT)
      SUM=0.
      DO 330 I=1,NT
  330 SUM=SUM + X(I)**B
      C(2,2,KOUNT)= 2.*SUM/A**3 -N/A**2
      P(1,KOUNT)=B
      P(2,KOUNT)=A
      CALL INVERT(C(1,1,KOUNT),2,E,DET)
      PRINT 6970
 6970 FORMAT(1H0,10X,25H COHEN*S VAR.-COV. MATRIX)
      PRINT 6971
 6971 FORMAT(1H0,9X,5HSHAPE,15X,5HSCALE)
      PRINT 1020,((C(I,J,KOUNT),I=1,2),J=1,2)
C
C  ESSENWANGER*S PARAMETERS AND VARIANCE-COVARIANCE MATRIX
```

```
C  ARE CALCULATED.
C
       CALL GAMMA((1.+1./B),R,$119,$150)
       GO TO 118
 119    R=ALOG(R)
 118    AMEAN=R*A**(1./B)
       ONE=1.
       DA=A
       DB=B
       DDA=DA+1.E-5*DA
       DDB=DB+1.E-5*DB
       CALL GAMMA((ONE+ONE/DB),R1,$120,$150)
       GO TO 121
 120   R1=ALOG(R1)
 121   DMEAN=R1*DA**(ONE/DB)
       CALL GAMMA((ONE+ONE/DDB),R,$122,$150)
       GO TO 123
 122   R=ALOG(R)
 123   F(1)=(R*DA**(ONE/DDB)-DMEAN)/(DB*1.E-5)
       F(2)=(R1*DDA**(ONE/DB)-DMEAN)/(DA*1.E-5)
       GO TO 171
 150   WRITE(6,151)
 151   FORMAT(1H ,10X,28HGAMMA VALUE NEGATIVE OR ZERO)
 171   CONTINUE
       EMEAN=C(1,1,KOUNT)*F(1)**2+2.*C(1,2,KOUNT)*F(1)*F(2)+
     .  C(2,2,KOUNT)*F(2)**2
C   AP IS THE PARTIALS OF ESSENWANGER*S PARAMETERS W.R.T. COHEN*S
       AP(1,1)=1.
       AP(1,2)=0.
       AP(2,1)=-ALOG(1./B)*(A**B)/(B*B)
       AP(2,2)=(A**(1./B -1.))/B
C   421 LOOP PREMULTIPLIES C BY AP
       DO 421 I= 1,2
```

```
      DO 421 J= 1,2
      APT(I,J)=0.
      DO 421 K=1,2
  421 APT(I,J)=APT(I,J)+ AP(I,K)*C(K,J,KOUNT)
C   422 LOOP POST MULTIPLIES APT BY AP TRANSPOSE
      DO 422 I=1,2
      DO 422 J=1,2
      ACV(I,J)=0.
      DO 422 K=1,2
  422 ACV(I,J)=ACV(I,J)+APT(I,K)*AP(J,K)
      PRINT 6980
 6980 FORMAT(1H0,10X,31H ESSENWANGER*S VAR.-COV. MATRIX)
      PRINT 6971
      PRINT 1020, ((ACV(I,J),I=1,2),J=1,2)
C
C  THIS CONVERTS COHEN'S SCALE PARAMETER TO ESSENWANGER'S SCALE
C  PARAMETER.
C
      ALPHA=EXP(ALOG(A)/B)
      CALL GAMMA((1.+2./B),VX1,$126,150)
      GO TO 127
  126 VX1=ALOG(VX1)
  127 CONTINUE
      CALL GAMMA((1.+1./B),VX2,$129,$150)
      GO TO 130
  129 VX2=ALOG(VX2)
  130 CONTINUE
      VX=(VX1-VX2**2)*ALPHA**2
      PRINT 997, AMEAN  ,EMEAN
 997  FORMAT(32H      EXP(X) - THEORETICAL MEAN =,
     1F15.5,26H            VARIANCE MEAN = ,F15.5)
      PRINT 6969, VX
 6969  FORMAT(13H     VAR(X) =,F15.5)
 1020 FORMAT(//(2E20.7)//)
```

```
C      A IS COHEN*S SCALE PARAMETER
C      B IS ESSENWANGER'S SHAPE PARAMETER (IT IS ALSO COHEN'S SHAPE
C      PARAMETER - ESSENWANGER'S AND COHEN'S SHAPE PARAMETER ARE
C      ALWAYS EQUAL).
C      ALPHA IS ESSENWANGER*S SCALE PARAMETER
       ALH(KOUNT)= ALPHA
       BEH(KOUNT) = B
       ME(KOUNT) = AMEAN
       VAX(KOUNT) = VX
       VAXH(KOUNT) = EMEAN
       PRINT 1030, B,ALPHA
1030   FORMAT(///10X,28HMAXIMUM LIKELIHOOD ESTIMATES/10X,13HSHAPE(BETA) =
      1,F15.7,19H    SCALE(ALPHA) =,F15.7,///)
C      THIS IS THE PLOT LOOP
       PRINT 1069
1069   FORMAT(1H0,12X,2H X,18X,6H PDF X,14X,6H CDF X,//)
        AINV = X(N)/80
        Y=0
       DO 333 I=1,100
        Y = Y + AINV
       Z(I,1)=Y
       Q=(2.7182818)**(-(Y**B)/A)
       Z(I,2)=Q*(B/A)*((Y)**(B-1.))
       T1=0.
       T2=0.
       K1=1
       K2=N+1
       DO 331 K=K1,N
       IF(X(K).GT.Y) GO TO 332
       T1=T1+1.
331    CONTINUE
332    Z(I,3)=T1/NT
       K1=T1
```

```
      IF(N.EG.NT) GO TO 336
      DO 335 K=K2,NT
      IF(X(K).GT.Y) GO TO 336
      T2=T2+1.
  335 CONTINUE
  336 Z(I,4)=(T1+T2)/NT
      K2=T2
      Z(I,2)=1.-Q
      IF(Z(I,2).EG.1.0) GO TO 333
      PRINT 654, Z(I,1),Z(I,5),Z(I,2)
  654 FORMAT(1H ,3F20.5)
  333 CONTINUE
      PRINT 915
  915 FORMAT(1H1,30H   CUMULATIVE DENSITY FUNCTION)
      CALL PLOT(KOUNT,Z,100,4,100,1)
      PRINT 1070
 1070 FORMAT(1H1,20H   DENSITY FUNCTION)
      DO 4444 I =1,100
 4444 Z(I,2)=Z(I,5)
      CALL PLOT(KOUNT,Z,100,2,100,1)
      RETURN
      END
```